Tracing Geometry

by Catheryne Draper

Good math is memorable math**,** the kind that stays with students so that they can remember it and apply it to new situations. Many students have a significantly better chance of remembering if they can "see" the math and then recognize the connections among topics. It isn't only about mastering the pieces or the skills; it's about *seeing* how the pieces and skills fit together. Our job as teachers is to help students see and hopefully remember valuable mathematical relationships from one grade level to the next. The chances for a student to see what we are teaching are significantly better when the student is looking at the same thing that we are teaching about. This idea seems so very obvious. Unfortunately it is a larger challenge than we teachers sometimes realize.

One of my favorite and most effective math tools across all grade levels is neither a ruler nor a protractor. It is tracing paper, or *patty paper* (the sheets that the food industry places between beef patties, hence the name). When students trace a shape, a math symbol, or an illustration of a quantity while you are teaching about that shape, symbol, or quantity, then you can be sure of two things: One, the students are looking at the same item or components of the item you are teaching about, and two, the students are actively participating in the lesson. The extent of student involvement after the tracing is up to you.

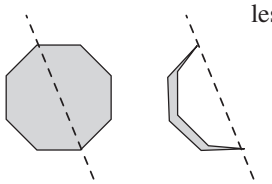
Students can use tracing paper or patty paper to make coin rubbings.

One of my early experiences with the benefits of patty paper was in a second-grade class during my lesson on money, specifically the identification of coins. The students did "coin rubbings," to help them focus on both the size and the imprint of the coin face. I was delighted with their reactions and the conversations with the students that this activity prompted.

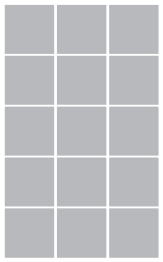
A less dramatic, yet equally effective, experience for another lesson at a different grade level was an activity involving shape properties. Students traced the shapes as assigned—trapezoids, rhombi, circles, squares, parallelograms, and rectangles—and were asked to fold the papers to illustrate diagonals, symmetry, and any side lengths that were the same measure. Because that was so successful, I mixed the regular polygons in with the irregular polygons so that students could identify more properties and specify when the properties would be applicable. I have used this activity at several grade levels, depending on the lesson focus.

Eventually I started a growing file of appropriate lessons that include the use of patty paper either as an introduction or as an exploration for further discussion. Since then I have also reviewed many publications that list other teachers' experiences with this material and attended workshops when teachers have incorporated patty paper into the sessions. A sampling of some of the activities that I have created, borrowed, and used with students is listed in the table below and I have correlated the activity to the recently distributed National Council of Teachers of Mathematics (NCTM) publication *Curriculum Focal Points for Pre-Kindergarten through Grade 8 Mathematics*.

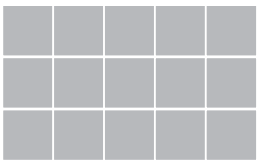
The activities in this table describe only a starter set of things that students can trace as an active part of your lessons. The references from the *Curriculum Focal Points* show how tracing and discussing can support the *NCTM Focal Points* and the *Principles and Standards for School Mathematics.*

Things to trace	Student activity	Corresponding Focal Point
Trace regular shapes square, rectangle, triangle, pentagon, trapezoid, rhombus, circle, others. 	Fold the shapes to identify lines of symmetry or diagonals or properties that are appropriate to the grade level lesson	Grade 1, Geometry: *…Initial understandings of … congruence and symmetry*. (p.13) Grade 3, Geometry: *…Describe, analyze, compare, and classify two-dimensional shapes…* (p. 15).
Trace irregular shapes	Reflect the shapes across a line of symmetry or rotate the shapes about the center to identify what happens when: a) The shapes are regular or b) The shapes are irregular Find lines of symmetry and diagonals and recognize which properties make a shape symmetrical, etc.	Grade 4, Geometry: *… Extend understanding… find areas of polygons and … use transformations.* (p. 16)
Trace rectangles on a multiplication table, starting from the upper left-hand corner	Notice and discuss that the lower right hand number describes the area of the rectangle. Rotate the rectangle 90 degrees and find the new area number in the chart.	Grade 3, Number and Operations: *… Use [geometric] properties … of multiplication.* (p. 15)

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

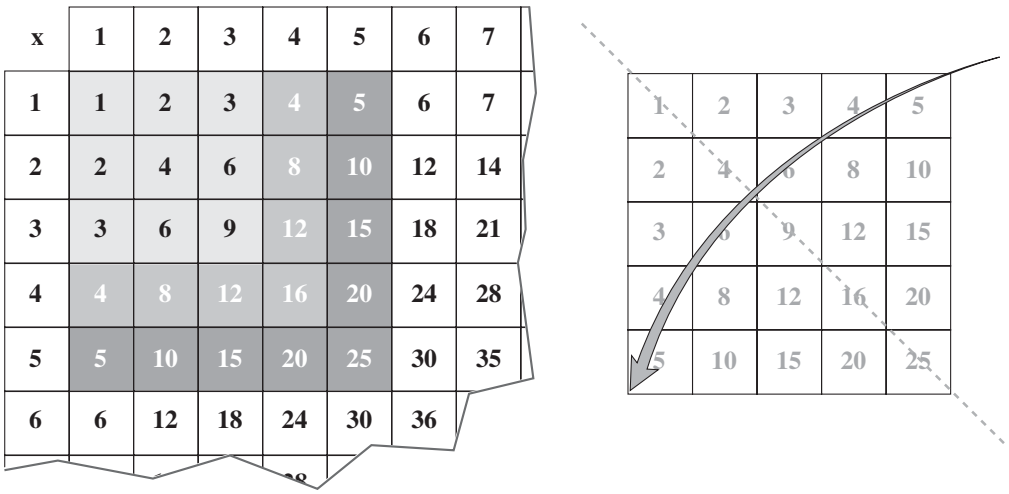


5 x 3



3 x 5

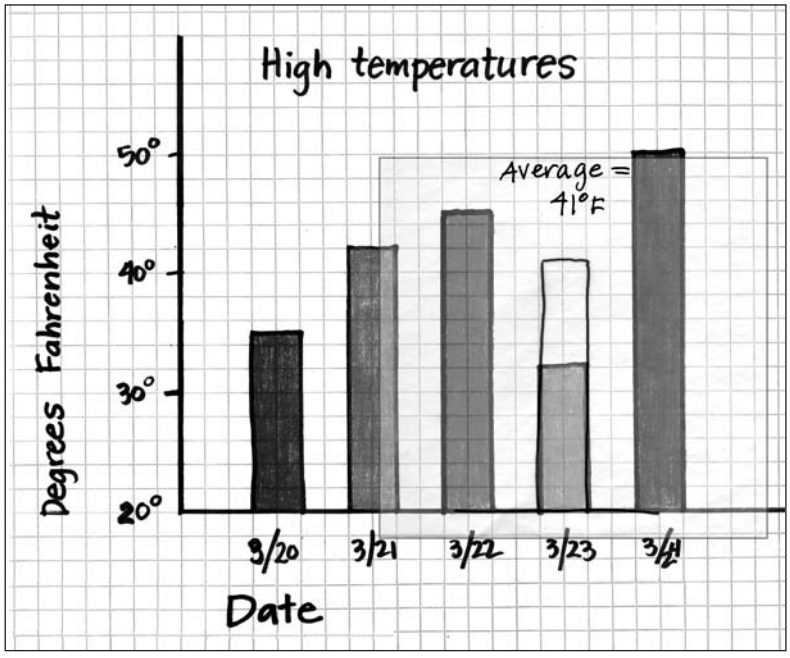
Things to trace	Student activity	Corresponding Focal Point
Trace squares on a multiplication table, starting from the upper left-hand corner	Fold the square tracing by bringing the upper right hand corner down to meet the lower left hand corner. What do you notice about the numbers that the resulting diagonal crease bisects?	Grade 4, Measurement: *… Connect area measure to area model…use the connection to justify the formula….* (p. 16)

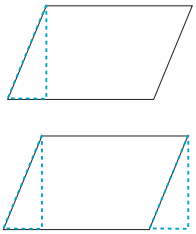


x	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	8	10	12	14
3	3	6	9	12	15	18	21
4	4	8	12	16	20	24	28
5	5	10	15	20	25	30	35
6	6	12	18	24	30	36	

Things to trace	Student activity	Corresponding Focal Point
On a bar graph, trace the bar that represents the average	Use the tracing of the average quantity to compare with each bar on the graph. Talk about the differences between their heights. By how much would each bar height need to change to match the average?	Grade 8, Data Analysis: *… Compare the information provided by the mean….* (p. 20)

Cathy Draper has more than forty years experience in math education. She has also worked as a math textbook editor and math supplementary materials editor. She started The Math Studio in the early 80's and published, "The Algebra Game," and, "Solving with Pythagoras," for cooperative learning groups. Visit the website at http://www.mathstudio.com.



Things to trace	Student activity	Corresponding Focal Point
Trace parts of areas of geometric shapes 	Rearrange and reassemble the parts. Discuss such questions as, "Has the area changed?" " Is the perimeter different?" (Many texts ask students to cut off the triangle so that the new shape is clearly a rectangle. This is a good idea but it does not allow for discussion about the shape that is now a trapezoid.)	Gr. 3, Geometry: *… Investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons.* (p. 15)

Things to trace	Student activity	Corresponding Focal Point
Trace a linear graph of an equation on a coordinate axis	Rotate graph of linear equation at a given point on the line until the traced line is perpendicular to where it was. Write the equation of the new line, using the same x and y axis as the original line uses. Slide the line above or below the y axis, keeping the angle with the y axis parallel to the original line. Write the equations for the new line using the same x and y axis as the original line.	Gr. 8, Algebra: *… Use linear function, linear equations, and systems of linear equations….* (p. 20)

Full involvement with learning

Several years ago I was in a workshop where a reference about retention was discussed. It was said that William Glasser reminds us that students forget 70% to 90% of information within the first 18 to 24 hours. He adds that students remember about 10% of what they read. The percentage increases to 30% of what students see and this increases to 50% of what students see *and* hear. The percentage is estimated to rise to as much as 80% when students experience the learning personally. Imagine remembering 80% of what you learn! If we teachers can unite the students' experiential involvement with the power of seeing the pieces, then we clearly have a winning combination.

Resources

William Glasser Institute, http://www.wglasser.com/.

Sawyer, W.W., *Vision in Elementary Mathematics*. Penguin Books, 1964.

Sawyer, W.W. *Mathematician's Delight*. Penguin Books, 1943.

Serra, Michael. *Patty Paper Geometry*. Key Curriculum Press, 1994.

The National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. NCTM, 2000.

The National Council of Teachers of Mathematics. *Curriculum Focal Points*. NCTM, 2006.